Functions

Math 130 - Essentials of Calculus

6 September 2019

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3. $h(x) = \dfrac{\sqrt{x}}{x^2 - 3x + 2}$

Vertical Line Test

Theorem

A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the graph more than once.

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4. $k(t) = 1 - t + t^2$

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4. $D(x) = f(x)/g(x)$

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Example

The annual revenue of a small store, in thousands of dollars, is given by $R(t) = 645 + 21t$, where t is the ear, with $t = 0$ corresponding to 2000. Similarly, the store's annual profit is given by $P(t) = 175 + 16t - 0.3t^2$.

1. *Write a formula for the annual cost function $C(t)$ for the store.*
2. *Compute $C(3)$ and interpret the result in this context.*

Combining Functions

Another way to combine functions is by composition.

Definition

Given two functions f and g, the composition of f and g is defined by

$$h(x) = f(g(x)).$$

Essentially, this is just putting the output of g in as an input of f.

Example

Let $f(x) = x^2 + 1$ and $g(t) = 4t - 2$. Let $h(t) = f(g(t))$ and $k(x) = g(f(x))$.

1. *Compute $h(3)$, then find a formula for $h(t)$.*
2. *Compute $k(3)$, then find a formula for $k(x)$.*

Decomposing Functions

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Example

Find functions f and g such that $h(x) = f(g(x))$.

1. $h(x) = \sqrt{x^3 - 1}$
2. $h(x) = \dfrac{1}{x^2 - 5}$

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3. $y = f(x - c)$ translates the graph c units to the right

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4. $y = f(\frac{1}{c}x)$ compresses the graph horizontally by a factor of c

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1. $y = -f(x)$ reflects the graph of $y = f(x)$ about the x-axis
2. $y = f(-x)$ reflects the graph of $y = f(x)$ about the y-axis

Graphing Transformations

Example

Sketch a graph of $f(x) = x^2$. Then graph the following transformations of $f(x)$.

1. $y = 2f(x)$
2. $y = f(x) - 3$
3. $y = \frac{1}{4}f(x) + 2$